OPTIMIZING POWER GENERATION USING BLACK BOX SPECIFICATION METHOD

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ABSTRACT

The efficiency of power generation is dependent upon the equipment selected during initial project engineering and is generally a multivariable function of several parameters. Therefore, conventional methods to optimize the power generation are typically applied after project engineering has completed the selection and specification of all necessary equipment.

The conventional method of specifying new power plants is to select the rated point as the best efficiency point for those parameter values, which occur most frequently during the selected time period. If the power plant operates off the rated point, the efficiency is reduced.

The disadvantage of conventional methods is that no optimization method can be applied to the process of specification and selection itself. Conventional methods optimize equipment, which has already been specified and selected.

To eliminate this disadvantage, a method is presented to optimize the power generation before project engineering has selected any specific equipment. The method is applicable to unknown equipment and to unspecified parameters of power generation, using a “Black Box Model” for the power generation equipment.

The presented “Black Box Specification Method” is an algorithmic procedure to optimize the power generation through specification of the rated point for single or multiple input parameters of any known or unknown equipment design.

Furthermore the efficiency of power generation depends on several parameters, which vary over certain periods of time, for example during one week or one year. Power plants are also subject of an aging process over several years with an observed shift of the parameter values.

The extended “Black Box Specification Method” applies Operations Research to specify the rated point such that the overall power plant efficiency is maximized for varying parameters over a certain period of time, taking further into account inevitable efficiency changes due to the aging process of the power plant equipment.

POWER GENERATION

Any kind of power generation has a certain input and output value of power. The ratio between the output and input value is the efficiency $\eta$ of power generation. The solid line graph of Figure 1 shows a typical efficiency curve for power generation. The efficiency $\eta$ is a multi-dimensional function of several input parameters $p_x$. For any of the input parameters $p_x$ the efficiency reaches its maximum value, the best efficiency point (BEP), for a certain value of the input parameter $p_x$.

$$\eta = f ( p_1, p_2, \ldots, p_x, \ldots, p_n )$$

By selecting one variable parameter $p_x$ and keeping all other parameters constant, the efficiency curve reaches the optimum (BEP) for the selected parameter when

$$\frac{\partial \eta}{\partial p_x} = 0$$

The actual curve and the location of the BEP are unknown, as described in the introduction, but it is ascertained that an optimum exists within a defined range of the parameter.

The simultaneous solution of this set of equations1 for all $n$ independent parameters $p_x$ ($x = 1, 2, \ldots, n$) defines the condition for the Absolute Best Efficiency Point (ABEP).

In hydroelectric power plants, for example, the input value is the available hydraulic power and the output value is the generated electrical power. Other input parameters are the volumetric flow of water, the height difference between upper and lower water reservoir, the water density and temperature, and equipment parameters like rotational speed of turbine and generator.

For hydraulic power recovery turbines used in the petroleum industry2, chemical industry or in cryogenic gas liquefaction plants1,1 the input parameters are the pressures in the high and low pressure vessels, the volumetric flow, the fluid density, the pressure losses across the control valves, and other equipment parameters.
The multidimensional efficiency function is an unknown function and the conventional method to analyze unknown functions is to expand a generally presumed function into a Taylor polynomial of a certain degree. A Taylor polynomial of quadratic degree offers an acceptable approximation, particularly if it is expanded at the optimum.

In Figure 1 the dotted curve demonstrates a quadratic Taylor polynomial which is expanded at the BEP for one selected parameter $p_x$. Because of the above described condition for the BEP, the first derivative of the efficiency function or the linear term of the Taylor polynomial is zero, and the Taylor polynomial is reduced to

$$\eta = \eta_{ABEP} - \sum C_x (p_x - q_x)^2$$

(Summation over all $x$, and with $\eta = \eta_{ABEP}$ for $p_x = q_x$)

$\eta_{ABEP}$ = unknown efficiency at the ABEP
$C_x$ = unknown constant
$p_x$ = input parameter
$q_x$ = unknown location of the ABEP

The quadratic Taylor polynomial approximates the efficiency function with a multidimensional normalized paraboloid with quadratic terms only. All mixed terms are negligible and may be disregarded.

The input parameters $p_x$ vary over a period of time, for example during one year, and the time $t$ can be expressed as a function of $p_x$

$$t = t(p_x)$$

It is assumed that all functions $t(p_x)$, the time distributions of the input parameters over a certain period of time are known. For example the volumetric flow of water in hydroelectric power plants over a period of one year can be predicted from statistical material.

**DISCRETE PARAMETER POINTS**

In the first case it is assumed the input parameters $p_x$ are discrete and have a certain value $p_{xy}$ for a certain period of time $t_y$. The total sum of all time values $t_x$ is set to be equal to unity 1.

To optimize power generation efficiency through specification, it is shown by Habets and Kimmel that the specified rated point of the projected power plant can be determined without the knowledge of the selected equipment through the following algorithm:

$$q_x = \sum t_{xy} p_{xy}$$

(Summation over all $y$)

The value of $q_x$ is the time-weighted arithmetic mean of the parameters $p_{xy}$.

The location of the specified rated point for optimized power generation is the location of the Absolute Best Efficiency Point (ABEP).

Figure 2 illustrates the method for optimal specification for discrete parameter points of one selected parameter $p_x$. The dotted curve shows the conventional specification method for the rated point, wherein the parameter point with the highest time value $t_{xy}$ is identical to the specified rated point and to the best efficiency point. All other parameter points are off the BEP on the declining part of the efficiency curve.

The solid line curve demonstrates the optimized specification. The rated point, identical to the BEP, is now between all parameter points and, in general, none of the parameter points is directly located at the BEP.

The distribution of all parameter points is such that the summation of all efficiency values will be at a maximum value.

**CONTINUOUS PARAMETER DISTRIBUTION**

For the more general case the input parameters $p_x$ have a continuous distribution over a period of time $t$.

The total power output $P$ can be calculated with

$$E = \int \eta t(p_x) \, dp_x$$

($p_x$ ranging from $p_{xmin}$ to $p_{xmax}$, and $x = 1, 2, 3 \ldots n$;)

Using the Taylor polynomial for $\eta$, the equation for $E$ transforms into

$$E = \int \left[ \eta_{ABEP} - \sum C_x (p_x - q_x)^2 \right] t(p_x) \, dp_x$$

$E$ has an optimum for the condition

$$\frac{\partial E}{\partial q_x} = 0$$

(for each $x = 1, 2, 3 \ldots n$)

which reads
\[ \frac{\partial E}{\partial q_x} = 2C_s \int (p_x - q_x) t(p_x) \, dp_x \]

The condition is satisfied for
\[ q_x \int t(p_x) \, dp_x = \int p_x \, t(p_x) \, dp_x \]

The integral over the time distribution is the total time
T of the selected period and is set to be equal to unity 1.
\[ T = \int t(p_x) \, dp_x = 1 \]

Analog to the case for discrete parameter points, the
specification method to optimize power generation for
continuous parameter distribution specifies the rated point
of the projected power plant without the knowledge of the
selected equipment.
\[ p_x_{\text{max}} q_x = \int t(p_x) p_x \, dp_x \]
\[ p_x_{\text{min}} \]

The value of \( q_x \) is the time-weighted arithmetic mean
of the continuous distribution of the parameters \( p_x \). The
location \( q_x \) of the specified rated point is the location of the
Absolute Best Efficiency Point (ABEP).

Figure 3 illustrates the optimal specification for the
continuous distribution of one selected parameter \( p_x \). The
dotted curve shows the conventional specification method
for the rated point, wherein the parameter point with the
highest time value \( t \) is selected to be the specified rated
point and also the best efficiency point (BEP). All other
parameter points are off the BEP on the declining part of the
efficiency curve.

The solid line curve demonstrates the optimized
specification. The rated point, identical to the BEP, is now
between all parameter points and the total power output over
the time period \( T \) will be a maximum value, independent of
the final equipment selection for the power generation.

The time distribution of the parameter is shown in
Figure 3 with the hatched area. The location \( q \) of the
specified rated point is the centerline of the hatched area. As
a result of the time-weighted arithmetic mean \( q \) the
negatively and positively hatched domains are equal in area.

If the input parameters of an unknown equipment for
power generation have a known time distribution, then the
power generation efficiency can be optimized by specifying
the rated point as the best efficiency point at the location of
the time-weighted arithmetic mean of the input parameters.

**AGING POWER EQUIPMENT**

All power generation plants and equipment are subject
of an aging process over several years. Due to wear,
corrosion and other aging causes, the BEP of the plant or
equipment is slowly moving out of the original location of
the rated BEP.

Figure 4 demonstrates the efficiency shift in aging
power generation equipment. The original BEP at the
location \( q_0 \) is the specified rated BEP without taking aging
processes into account. The corresponding original power
generation efficiency is the solid line curve. To simplify the
example, only one operation point for the input parameter \( p = q_0 \), the solid point, is shown. In this case the
operation point is identical with the BEP and the specified rated point,
but only if aging processes are disregarded. This is in
general the situation for a new power plant at the time of
commissioning.

Several years later the power equipment enter the
process of aging and the efficiency curve, together with its
BEP, starts shifting towards increased or decreased input
parameters, for example in hydroelectric power plants
towards higher flow rates or lower heads. The aged BEP is
then at the location \( p = q_\theta \), with \( \theta \) as the lifetime. The
original operation point \( p = q_0 \) is now operating along the
aged efficiency curve (the dotted curve in Figure 4) at much
lower efficiency with less power output, indicated with a
circle.

The aging process of power equipment is well known.
With the available experience of abundant existing power
plants, the effects of the aging process can be predicted and
therefore being used to optimize power plant specifications
incorporating the aging effect.

Figure 5 demonstrates the optimal specification for
aging power generation equipment. The aging time \( \theta \) is
graphed over the performance or input parameter \( p \). For a
new power equipment the aging time is \( \theta = 0 \), and \( \theta = \theta \)
at the time of decommissioning. The position \( q \) of the BEP is
now a function of the aging time \( \theta \), with \( q_\theta \) as final and \( q_0 \)
as final position and \( q_R \) as the rated BEP
\[ q = q_R + s(\theta) \]

The initial and final efficiency are graphed as broken
line curve and dotted curve in Figure 5. The shift \( s(\theta) \) in a
typical aging function is shown as the left border of the
hatched area.

The previous examples introduce \( t \) as the time variable
of the periodic parameter distribution for the time period \( T \).
for instance one months, whereas this chapter defines the aging time $\vartheta$ as the large scale time variable of the non periodic aging process of several years with the final age or lifetime $\vartheta = \theta$. The following condition holds

$$ T \ll \theta $$

The problem is to find the correct BEP location $q = q_R$ for which the sum and integral of all efficiency values is at optimum over all periods of time $T$ and over the aging time $\vartheta$ between $\vartheta = 0$ and $\vartheta = \theta$.

For a discrete parameter distribution $p_y$ with one parameter $p$ and a continuous aging process the sum $E$ of the integral of all efficiency values reads

$$ E = \sum \{ t_y \left[ \eta_{BEP} - C(p_y - (q_R + s(\vartheta))^2) \right] d\vartheta \}$$

With summation over all $y$ and integration from $\vartheta = 0$ to $\vartheta = \theta$. The condition for the optimum is the following equation

$$ \frac{\partial E}{\partial q_R} = 0 \text{ leading to } \sum t_y \left[ p_y - q_R - s(\vartheta) \right] d\vartheta = 0 $$

This condition transforms into

$$ \theta \sum t_y q_R = \theta \sum t_y p_y - \sum t_y \left[ s(\vartheta) \right] d\vartheta $$

to find the explicit solution for $q_R$

$$ q_R = \frac{\sum t_y p_y - \left[ \int s(\vartheta) d\vartheta \right] \theta^{-1}}{\theta} $$

The arithmetic mean $s_{am}$ of the shift function $s(\vartheta)$ is defined as

$$ s_{am} = \left[ \int s(\vartheta) d\vartheta \right] \theta^{-1} $$

The optimized specification of power plants which are subject to an aging process is defined with the rated point at the location $q_R$ and is determined by

$$ q_R = \sum t_y p_y - s_{am} $$

To take the aging process of power plants in advance into account, the rated point has to be specified as located to the opposite direction of the aging shift with a distance of the arithmetic mean of the shift function during the total lifetime.

Figure 5 shows for the initial operation the rated point $q_R$ as BEP of the broken line curve to the left of all efficiency curves, which will shift to the right side with progressing age.

The power generation efficiency over the total lifetime $0$ of the plant is an optimum if the rated point $q_R$ is specified using the above-presented algorithm.

**REFERENCES**

Figure 1. Typical efficiency curve for power generation

Figure 2. Optimal specification for discrete parameter points
Figure 3. Optimal specification for continuous parameter distribution
Figure 4. Efficiency shift in aging power generation equipment
Figure 5. Optimal specification for aging power generation equipment